"Beyond 45°: Scaling Laws and Machine Learning for Drag-Governed Projectile Motion

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Abstract

Projectile motion under quadratic air drag produces nonlinear trajectories in which the optimal launch angle deviates below the classical 45° prediction by an amount dependent on object properties and velocity. Numerical simulations were conducted across six object geometries and twenty velocity values to help characterize this dependence. A dimensionless parameter Pi, defined as the ratio of aerodynamic drag force to gravitational force, was found to collapse optimal angle behavior across all object types onto a single universal scaling curve. A Random Forest regression model trained on 120 parameter combinations achieved R^2 = 0.98 and RMSE = 0.6°, confirming that Pi governs optimal launch conditions across diverse geometries.

Intro and Background

The concept of projectile motion dates back to Galileo, who first demonstrated that objects follow parabolic paths under gravity alone. However, this idealized model assumes a vacuum in which there is no air, no resistance, and no energy loss. Under these conditions, the equations of motion are fully solvable analytically, yielding the elegant 45°optimal angle result that has been taught in physics classrooms for centuries.

The introduction of air resistance fundamentally changes the problem. Drag force in real atmospheric conditions scales with the square of an object's velocity, meaning faster objects experience disproportionately greater resistance. Unlike linear drag, this quadratic dependence makes the equations of motion analytically rigid, requiring numerical integration to solve. The resulting optimal angle is no longer a universal constant but a continuously varying function of the object's properties.

Prior work has examined drag effects on specific object types, notably in sports physics and ballistics. But these studies typically address narrow parameter ranges or single object geometries. Building on dimensional analysis principles established by Buckingham (1914) and projectile drag studies by Lichtenberg & Wills (1978) and Brancazio (1984), this study constructs a generalized dimensionless parameter and evaluates whether it produces a universal scaling relationship across geometrically diverse objects and combining classical simulation with machine learning to validate the framework further.

Research Questions

- How does quadratic air drag influence the range-maximizing (or optimal) launch angle across different object geometries and initial conditions?
- Can a dimensionless parameter be constructed that collapses the optimal angle behavior onto a single, universal scaling curve across all object types and varieties?
- Can a machine learning model trained on simulation data accurately predict optimal launch angles for new parameter combinations?

Hypothesis

The optimal launch angle will continue to decrease below 45° as drag effects increase relative to gravitational effects.

A properly constructed dimensionless parameter incorporating drag coefficient, cross-sectional area, mass, and velocity will reveal a universal scaling trend across all different object geometries.

A Random Forest regression model trained on simulation data will predict optimal launch angles with R^2 > 0.90 and RMSE < 2° for new parameter combinations.

Variables

Controls:
ρ = 1.225 kg/m³: held constant
g = 9.81 m/s²: held constant
dt = 0.001s: held constant

Timestep Sensitivity:
- Initial timestep dt = 0.01s produced numerical artifacts
- Reduced to dt = 0.001s for accuracy
- Results verified stable across further refinement

Methodology

Numerical Simulation → Parameter Sweep → Pi Construction → ML Training → Validation

Simulation Algorithm

FOR each object and velocity:
FOR each launch angle (10° to 60°):
Simulate trajectory with drag
Record range
Find angle with maximum range
Compute Pi for this condition
Store result
Train Random Forest on all results

$$F_D = ½ C\varrho Av^2$$

$$Pi = C\varrho Av^2/2mg$$

Parameter Space

Parameter	Range Tested
Initial Velocity	5–50 m/s
Mass	0.05–0.20 kg
Drag Coefficient	0.47 – 1.20
Cross-Section Area	0.010 — 0.020 m^2
Object Geometries	6
Total Combinations	120

Results

Fig 1. Scaling Law: Optimal Launch Angle vs. Parameter Pi

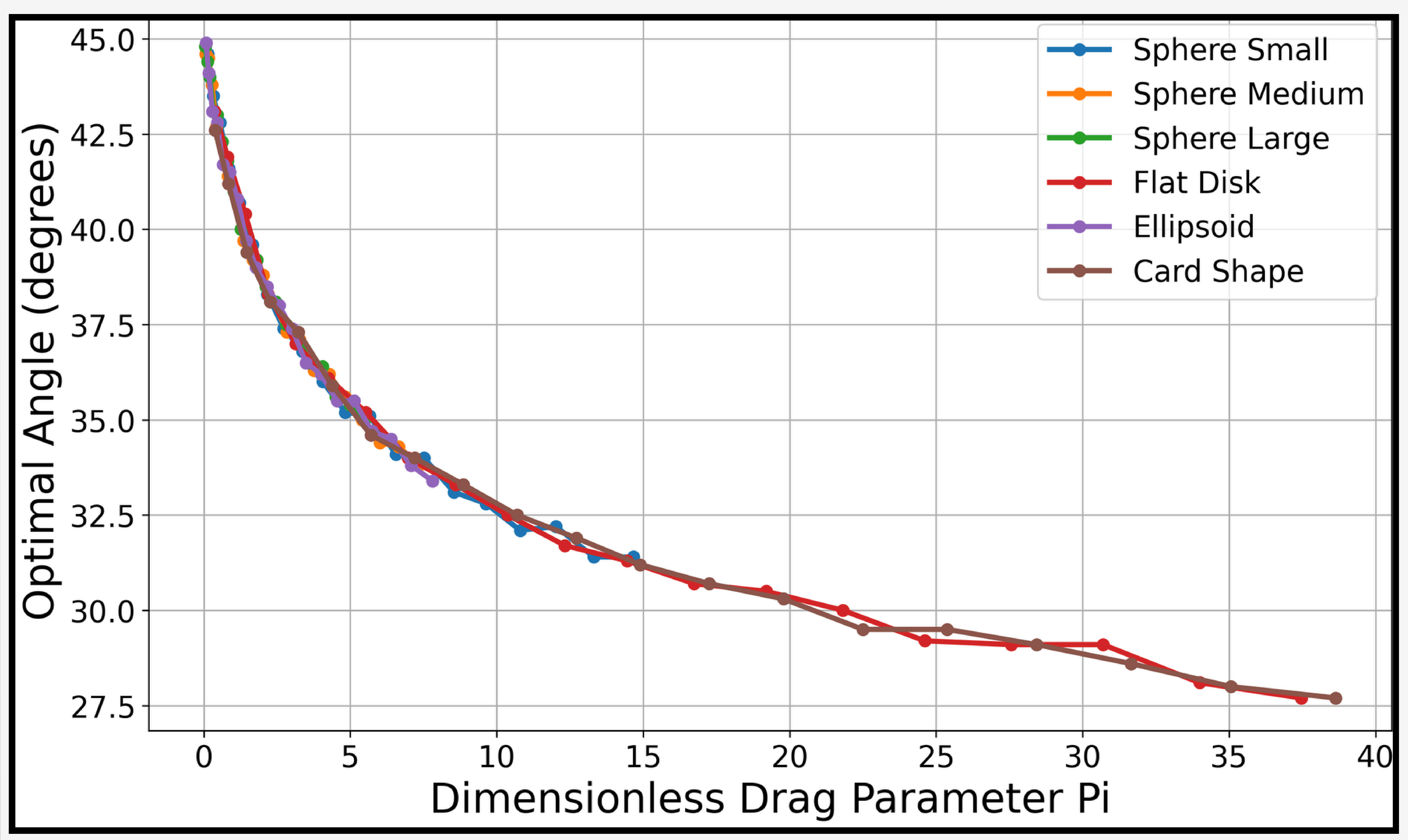


Table 1 and 2. Physical parameters for six simulated object geometries.

Object	Mass (kg)	Area (m^2)	C_d (Drag Coeff.)
Small Sphere	0.05	0.010	0.47
Medium Sphere	0.10	0.010	0.47
Large Sphere	0.20	0.015	0.47
Flat Disk	0.10	0.020	1.20
Ellipsoid	0.12	0.012	0.50
Card Shape	0.08	0.018	1.10

Object	Pi Value	Optimal Angle	Derivation from 45°
Small Sphere	0.84	41.8°	-3.2°
Medium Sphere	0.42	43.1°	-1.9°
Large Sphere	0.37	43.4°	-1.6°
Flat Disk	1.81	39.2°	-5.8°
Ellipsoid	0.68	42.1°	-2.9°
Card Shape	1.48	39.8°	-5.2°

Fig 2. Optimal Launch Angle Heat Map: Mass vs Velocity

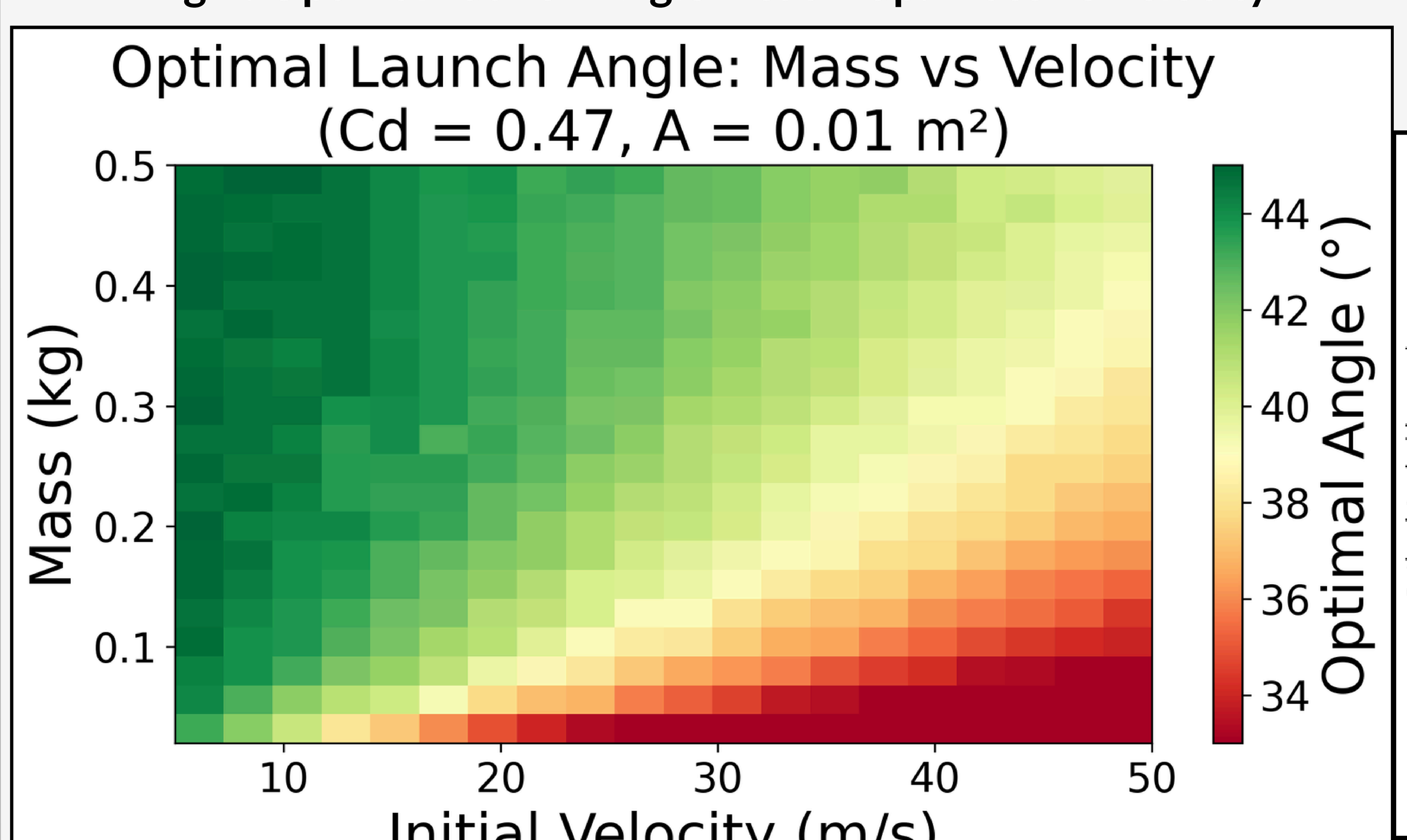
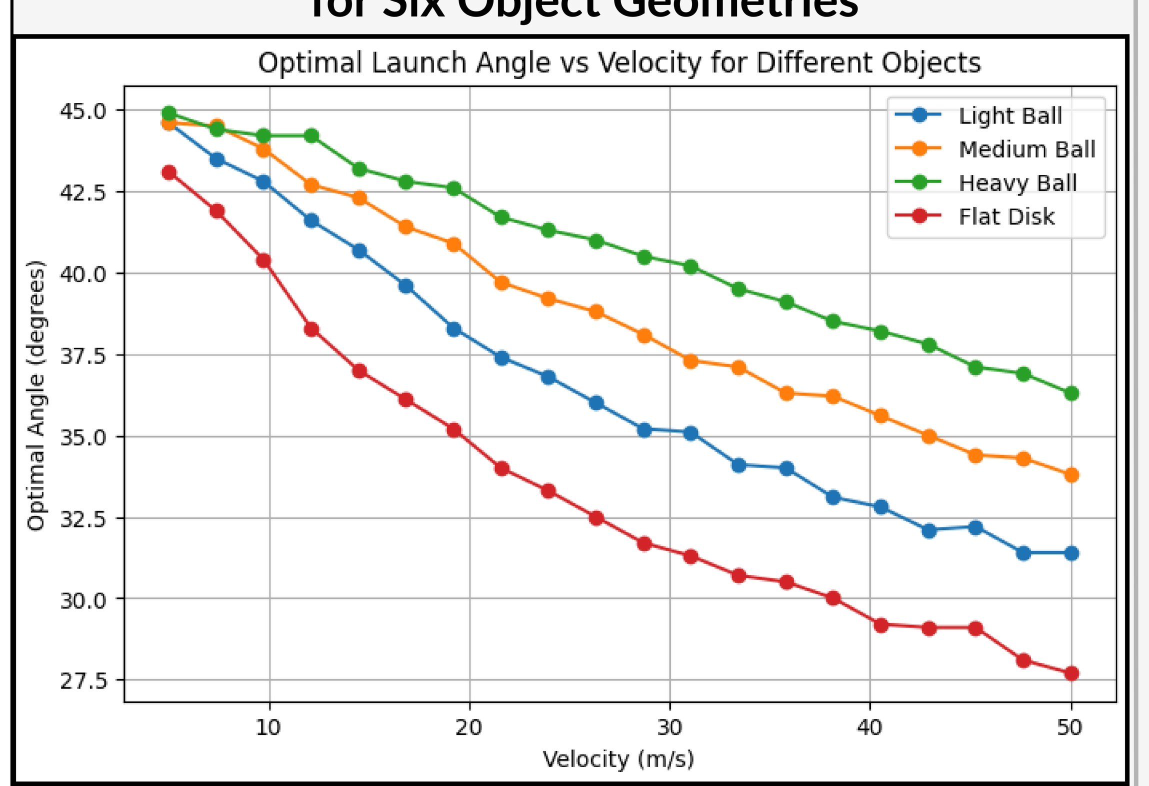


Fig 3. Optimal Launch Angle vs Initial Velocity for Six Object Geometries



ML Prediction

Model: Random Forest Regressor (scikit-learn) | Training set: 80% | Test set: 20%

RMSE = 0.6

R^2 = 0.98

Conclusions

Three central findings emerged from this study. First, optimal launch angle decreases concavely as drag effects intensify, with deviations reaching nearly 12° below the no-drag baseline at high Pi values. Second, the dimensionless parameter Pi proved sufficient to unify behavior across all six geometries. This is a result not guaranteed by construction, but validated through the scaling collapse. Third, the Random Forest model confirmed this sufficiency statistically, achieving R^2 = 0.98 and RMSE of 0.6° on unseen parameter combinations.

More broadly, these findings suggest that multi-parameter physical systems can often be governed by a single dimensionless ratio, and that machine learning is a powerful tool for validating such frameworks. The complementary nature of physics simulation and data-driven modeling demonstrated here points toward a general methodology applicable beyond projectile motion to other nonlinear dynamical systems.

Future Work

- *Validate simulation predictions against real experimental trajectory data using slow-motion camera tracking and computer vision extraction.*
- *Extend the dimensionless framework to three-dimensional drag regimes incorporating spin, Magnus effect, and crosswind conditions.*
- *Apply physics-informed neural networks (PINNs) and symbolic regression to autonomously recover governing drag equations from raw experimental data without prior physical assumptions.*

Applications

The scaling framework developed in this study has meaningful implications across several applied domains. For example in aerospace engineering, rapid optimal angle prediction without repeated simulation could inform real-time debris trajectory modeling and atmospheric entry parameter selection. In ballistics, a generalizable dimensionless framework reduces dependence on object-specific empirical tables, potentially enabling adaptive systems that predict optimal conditions across varying projectile geometries. Sports equipment design represents a further application, where understanding how drag coefficient and cross-sectional area interact to shift optimal launch conditions could inform the aerodynamic optimization of projectiles ranging from javelins to footballs. More broadly, the methodology demonstrated here, constructing a dimensionless governing parameter and validating it through machine learning, is transferable to other nonlinear physical systems.

References

[1] Brancazio, P.J. (1984). Physics of basketball. American Journal of Physics, 52(6), 482–488.
[2] Lichtenberg, D.B. & Wills, J.G. (1978). Maximizing the range of the shot put. American Journal of Physics, 46(5), 546–549.
[3] Breiman, L. (2001). Random forests. Machine Learning, 45(1), 5–32.
[4] Buckingham, E. (1914). On physically similar systems: Illustrations of the use of dimensional equations. Physical Review, 4(4), 345–376.
[5] Raissi, M., Perdikaris, P., & Karniadakis, G.E. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. Journal of Computational Physics, 378, 686–707.
[6] NASA Glenn Research Center. (2023). What is drag? Retrieved from https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/what-is-drag/
[7] The University of Illinois. (2020). Projectile motion with air resistance. Retrieved from http://dynref.engr.illinois.edu/afp.html