Beyond 45°

Scaling Laws and Machine Learning for Drag-Governed Projectile Motion

Naren Raghavan | Signature School — Evansville, Indiana

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Background & Motivation

1.
The Classical Result

In a vacuum, elementary mechanics yields a clean prediction: optimal launch angle is exactly **45°** for maximum range, regardless of object properties.

2.
The Real-World Problem

However, quadratic aerodynamic drag: $\mathbf{F = \frac{1}{2}\rho C d A v^2}$ breaks this symmetry. The optimal angle shifts downward in ways that depend on object type, velocity, and other properties.

3.
The Gap

As a result, no closed-form analytical solution exists. Prior work treats each object class separately, leaving no universal, predictive framework for drag-governed projectile motion.

Research Questions & Hypotheses

Research Questions

1. How does quadratic air drag influence the range-maximizing launch angle across different object geometries?
2. Can a dimensionless parameter collapse optimal angle behavior onto one universal scaling curve?
3. Can a machine learning model accurately predict optimal launch angles for unseen parameter combinations?

Hypotheses

H1
Optimal angle decreases monotonically below 45° as drag effects increase.

H2
A dimensionless parameter incorporating Cd, area, mass, and velocity reveals universal scaling across all geometries.

H3
Random Forest model predicts optimal angles with R^2 above 0.95 and RMSE below 2°.

Methodology

Numerical Simulation
RK4 integration of drag equations across 120 parameter combinations

Parameter Sweep
Varied mass, velocity, cross-section, and drag coefficient across six object classes

Pi Construction
$Pi = \rho C_d A v^2 / 2mg$

ML Training
Random Forest on 80/20 train-test split; Pi as primary feature

Validation
Cross-validated R^2 and RMSE against held-out simulation data

$$F\_drag = ½ C\varrho Av2$$

$$Pi = C\varrho Av\text{^}2/2mg$$

Parameter	Range Tested
Initial Velocity	5–50 m/s
Mass	0.05–0.20 kg
Drag Coefficient	0.47 – 1.20
Cross-Section Area	0.010 — 0.020 m^2
Object Geometries	6
Total Combinations	120

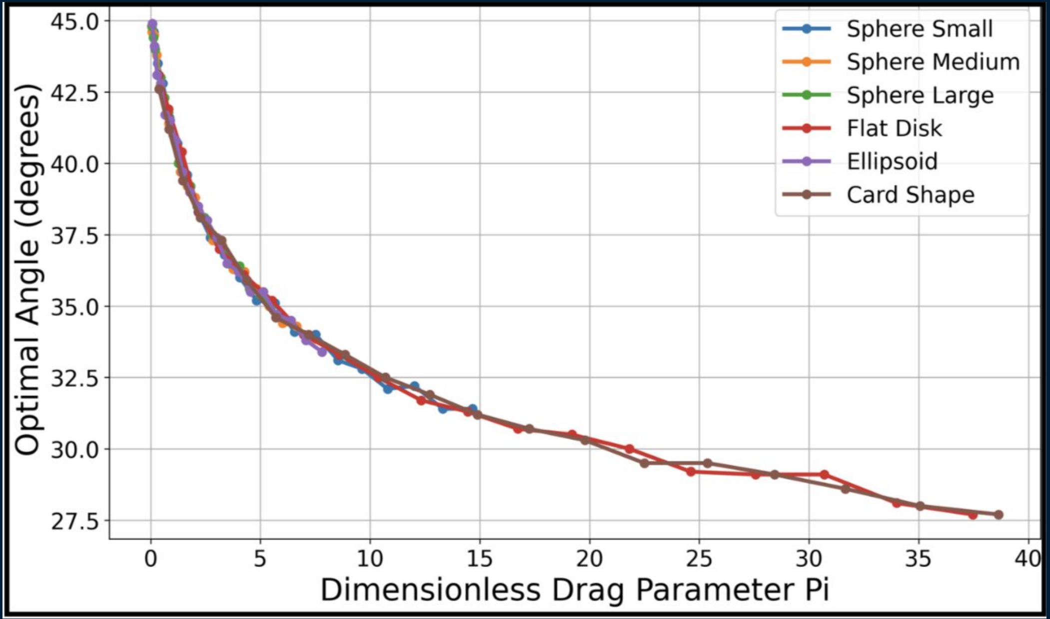
Controls: air density held constant at sea level (ρ = 1.225 kg/m³); launch height fixed at ground level; no wind or spin effects modeled.

Results II - Pi Scaling Collapse

The Key Discovery

When plotted against Pi instead of velocity, all six objects collapse onto one universal scaling curve.

- Pi alone governs optimal launch angle
    - Object type becomes irrelevant.
    - The dimensionless parameter fully encodes all physical complexity.

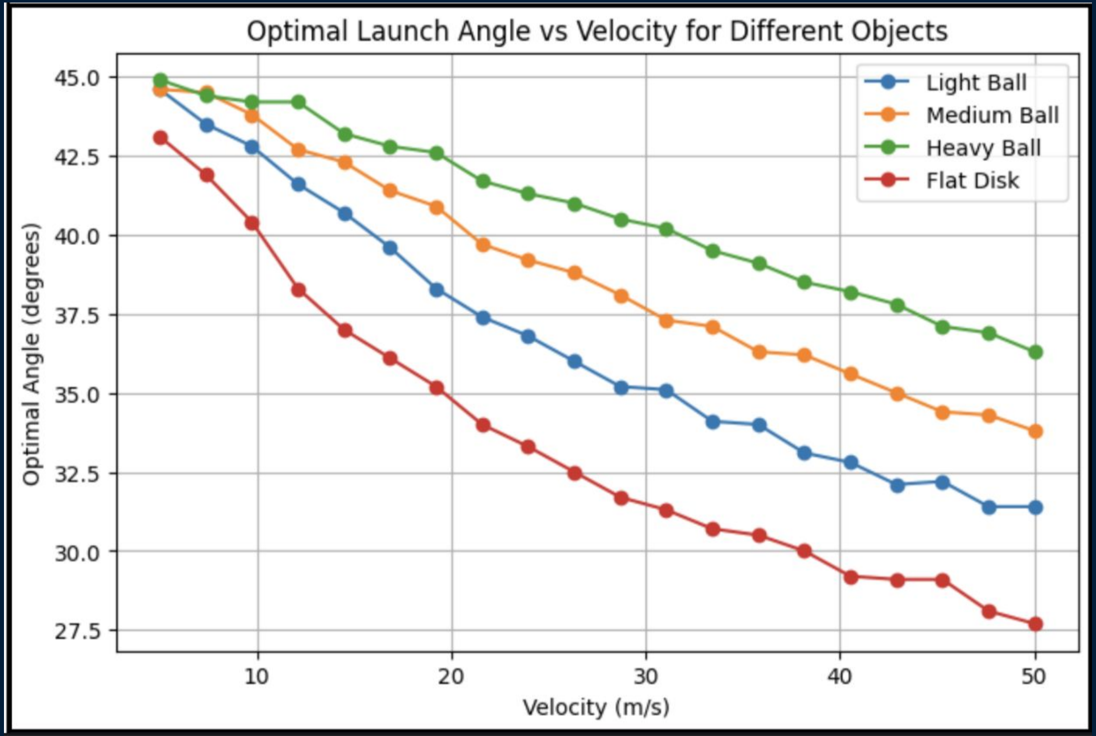


Results I - Optimal Launch Angle vs. Velocity

Key Observations

- All objects show optimal angle below 45° baseline.
- Higher velocity increases drag effect and lowers optimal angle.
- Six geometries produce six distinct decline rates.
- No single velocity-based rule unifies all objects.

Problem: six separate curves with no unifying relationship.



Machine Learning Validation

0.98

R^2 Score

Random Forest on held-out 20% test set

0.6°

RMSE

Root mean squared error in predicted optimal angle

Model Architecture

Random Forest (100 estimators) trained on Pi, mass, velocity, Drag Coefficient, and cross-sectional area.

Generalization Confirmed

The model successfully validated Pi through the metrics above and shows that Pi is crucial in governing the system.

Conclusions

H1 Confirmed

Optimal launch angle decreases concavely below 45° as drag effects increase, with deviations reaching nearly 12° at high Pi values.

H2 Confirmed

Dimensionless parameter Pi successfully collapses optimal angle behavior across all six geometries onto a single universal scaling curve.

H3 Confirmed

Random Forest model achieved $R^2 = 0.98$ and RMSE = 0.6°, accurately predicting optimal angles for unseen parameter combinations.

Broader Finding: Physics-based simulation and machine learning are highly complementary as simulation generates ground truth, ML validates the governing framework.

Future Work & Applications

1

Experimental Validation

Use of high-speed camera tracking of physical projectiles to validate Pi curves against real-world trajectories.

2

Extended Framework

Incorporate spin (Magnus effect), variable air density with altitude, and non-spherical geometries to test framework robustness.

3

Physics-Informed Neural Networks

Extend the current framework using PINNs that embed equations of motion directly, improving extrapolation beyond the training regime.

Application Domains

Aerospace Engineering

Rapid trajectory optimization for unpowered re-entry vehicles and guided munitions.

Ballistics & Defense

Field-deployable angle tables derived from Pi, replacing object-specific lookup charts with a single universal formula.

Atmospheric Entry

Pi framework extensible to entry vehicles in non-terrestrial atmospheres where object-specific data is unavailable.